

Dispersive Casimir interactions between atoms and surfaces

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Collaborators



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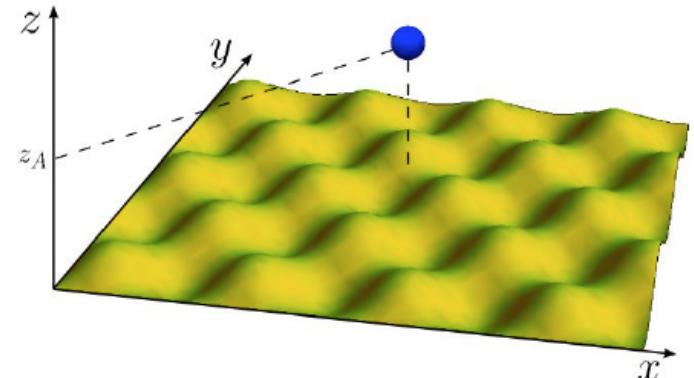
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Esteban Calzetta (Buenos Aires)

Outline of this talk

- Review of theory and experiment on Casimir atom-surface interactions



- Casimir-Polder forces within scattering theory

- Cold atoms for probing lateral Casimir-Polder forces

- BEC “cantilever” to measure Casimir frequency shifts

- BEC Bragg spectroscopy to measure Casimir energy profile

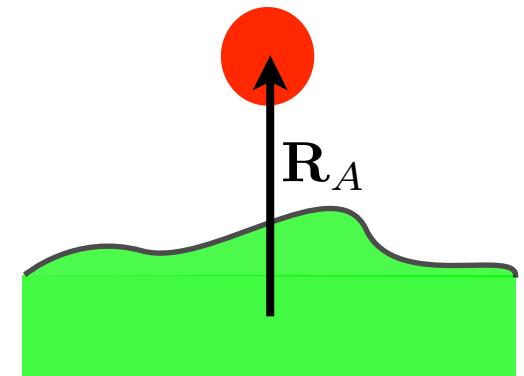
The Casimir-Polder force

vdW - CP interaction

Casimir and Polder (1948)

The interaction energy between a ground-state atom and a surface is given by

$$U_{\text{CP}}(\mathbf{R}_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \text{Tr } \mathbf{G}(\mathbf{R}_A, \mathbf{R}_A, i\xi)$$



Atomic polarizability: $\alpha(\omega) = \lim_{\epsilon \rightarrow 0} \frac{2}{3\hbar} \sum_k \frac{\omega_{k0} |\mathbf{d}_{0k}|^2}{\omega_{k0}^2 - \omega^2 - i\omega\epsilon}$

Scattering Green tensor: $\left(\nabla \times \nabla \times -\frac{\omega^2}{c^2} \epsilon(\mathbf{r}, \omega) \right) \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \delta(\mathbf{r} - \mathbf{r}')$

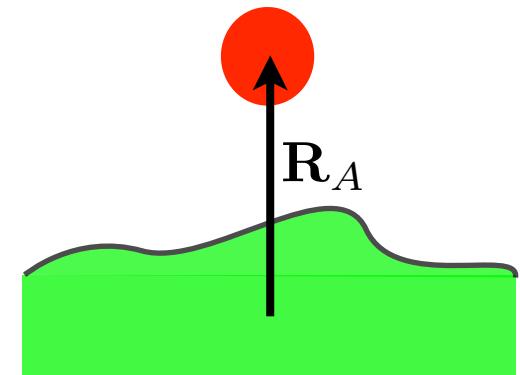
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■ vdW - CP interaction

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■ Eg: Ground-state atom near planar surface @ T=0

Non-retarded (vdW) limit $z_A \ll \lambda_A$

$$U_{\text{vdW}}(z_A) = -\frac{\hbar}{8\pi\epsilon_0} \frac{1}{z_A^3} \int_0^\infty \frac{d\xi}{2\pi} \alpha(i\xi) \frac{\epsilon(i\xi) - 1}{\epsilon(i\xi) + 1}$$

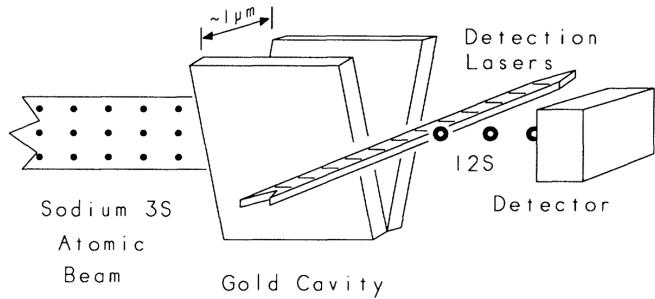
Retarded (CP) limit $z_A \gg \lambda_A$

$$U_{\text{CP}}(z_A) = -\frac{3\hbar c \alpha(0)}{8\pi} \frac{1}{z_A^4} \frac{\epsilon_0 - 1}{\epsilon_0 + 1} \phi(\epsilon_0)$$

Modern CP experiments



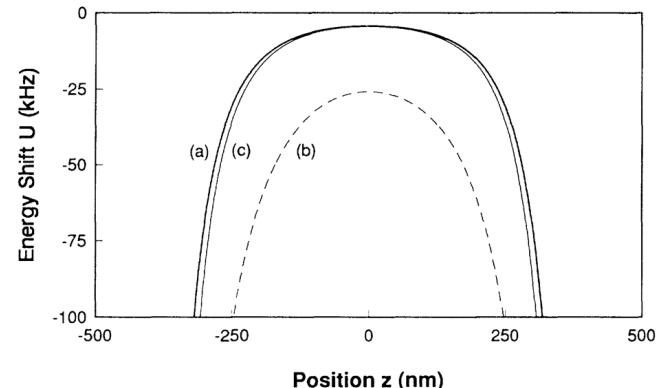
Deflection of atoms



$L = 0.7\text{-}1.2 \mu\text{m}$

Exp-Th agreement @ 10%

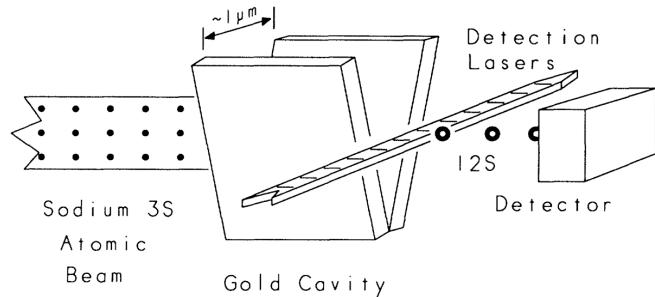
Hinds et al (1993)



$$U_{CP} = -\frac{1}{4\pi\epsilon_0} \frac{\pi^3 \hbar c \alpha(0)}{L^4} \left[\frac{3 - 2 \cos^2(\pi z/L)}{8 \cos^4(\pi z/L)} \right]$$

Modern CP experiments

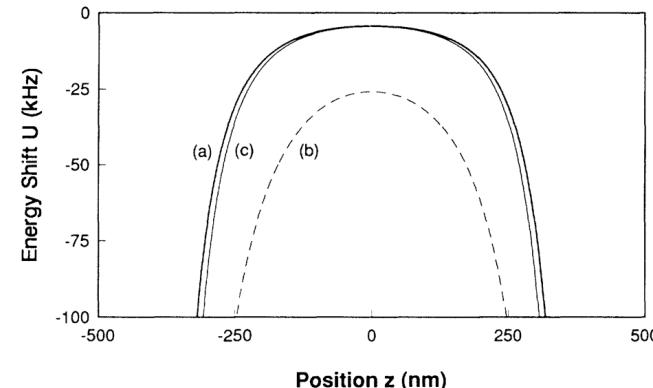
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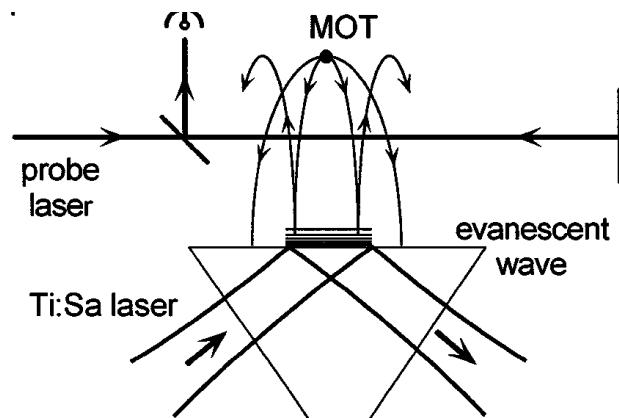
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Classical reflection on atomic mirror

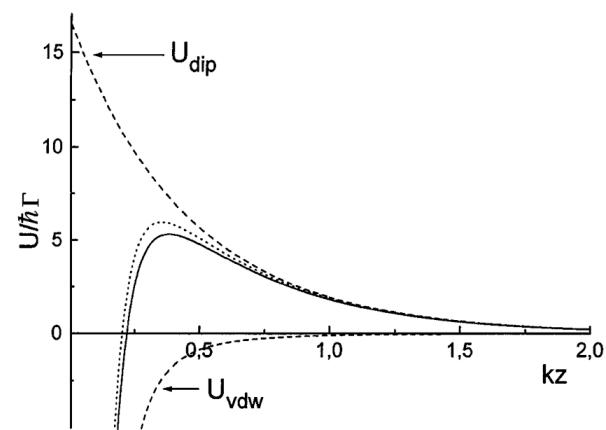


Exp-Th agreement @ 30%

Aspect et al (1996)

$$U_{\text{dip}} = \frac{\hbar}{4} \frac{\Omega^2}{\Delta} e^{-2kz}$$

$$U_{\text{vdW}} = -\frac{\epsilon - 1}{\epsilon + 1} \frac{1}{48\pi\epsilon_0} \frac{D^2}{z^3}$$

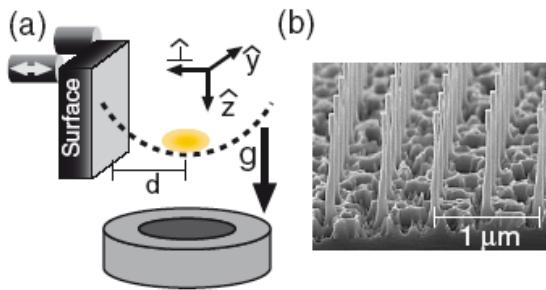


Modern experiments (cont'd)



Quantum reflection

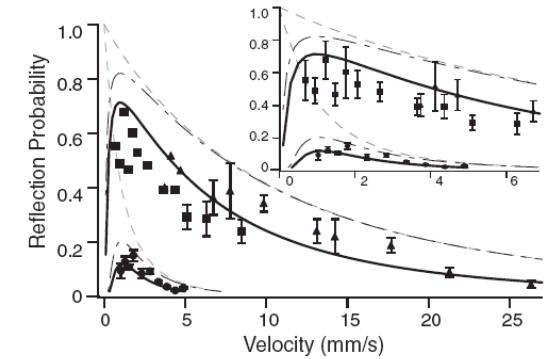
Wave-nature of atoms implies that slow atoms can reflect from purely attractive potentials



$$k = \sqrt{k_0^2 - 2mU/\hbar^2} \quad \phi = \frac{1}{k^2} \frac{dk}{dr} > 1$$

$$U = -C_n/r^n \quad (n > 2)$$

Shimizu (2001)
DeKievit et al (2003)



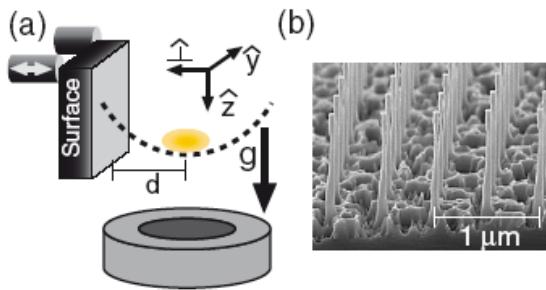
Ketterle et al (2006)

Modern experiments (cont'd)



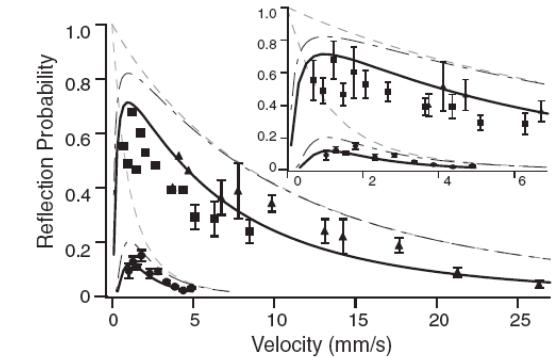
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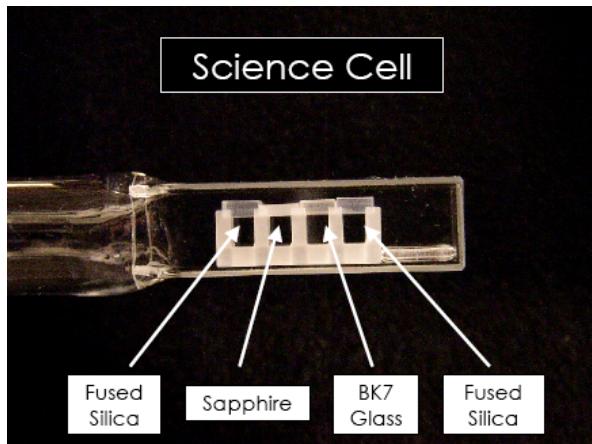
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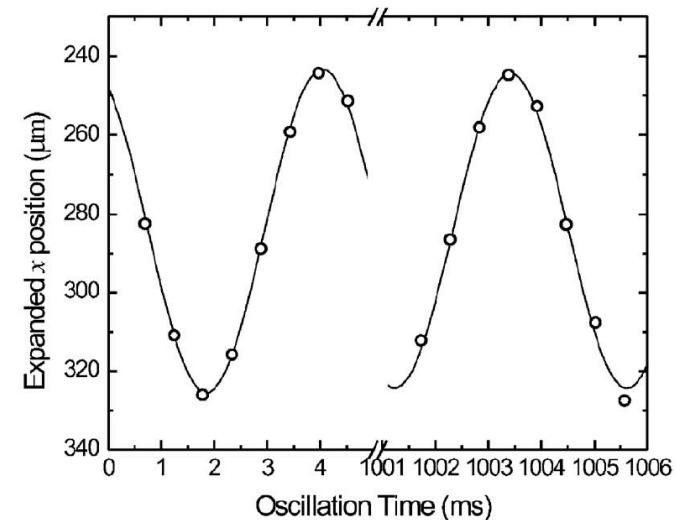


BEC oscillator

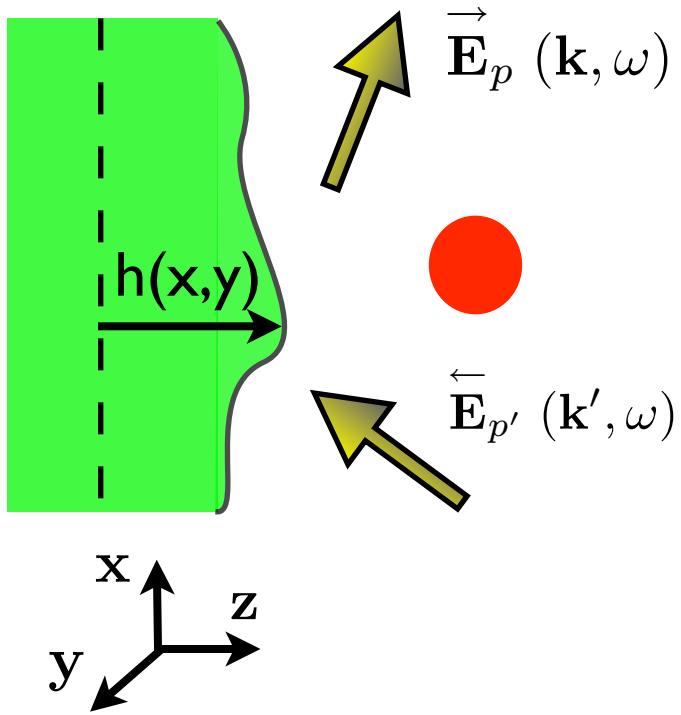


$$\gamma_x \equiv \frac{\omega_x - \omega'_x}{\omega_x} \simeq -\frac{1}{2\omega_x^2 m} \partial_x^2 U^*$$

Cornell et al (2007)



CP within scattering theory



Output fields:

$$\vec{E}(\mathbf{R}, \omega) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{-i\mathbf{k}\cdot\mathbf{r}} \vec{E}(\mathbf{k}, z, \omega)$$

$$\vec{E}(\mathbf{k}, z, \omega) = [\vec{E}_{TE}(\mathbf{k}, \omega) \hat{\epsilon}_{TE}^+(\mathbf{k}) + \vec{E}_{TM}(\mathbf{k}, \omega) \hat{\epsilon}_{TM}^+(\mathbf{k})] e^{ik_z z}$$

$$\hat{\epsilon}_{TE}^+(\mathbf{k}) = \mathbf{z} \times \mathbf{k} \quad \hat{\epsilon}_{TM}^+(\mathbf{k}) = \hat{\epsilon}_{TE}^+(\mathbf{k}) \times \mathbf{K} \quad (\mathbf{K} = \mathbf{k} + k_z \mathbf{z})$$

Input fields: idem with $k_z \rightarrow -k_z$

Input and output fields related via reflection operators

$$\vec{E}_p(\mathbf{k}, \omega) = \int \frac{d^2\mathbf{k}'}{(2\pi)^2} \sum_{p'} \langle \mathbf{k}, p | \mathcal{R}(\omega) | \mathbf{k}', p' \rangle \vec{E}_{p'}(\mathbf{k}', \omega)$$

Casimir-Polder force:

$$U_{CP}(\mathbf{R}_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \int \frac{d^2\mathbf{k}}{(2\pi)^2} \int \frac{d^2\mathbf{k}'}{(2\pi)^2} e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{r}_A} e^{-(\kappa+\kappa') z_A} \frac{1}{2\kappa'} \sum_{p,p'} \hat{\epsilon}_p^+(\mathbf{k}) \cdot \hat{\epsilon}_{p'}^-(\mathbf{k}') R_{p,p'}(\mathbf{k}, \mathbf{k}')$$

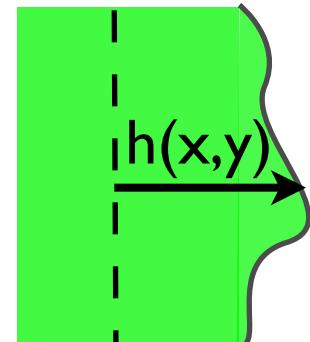
with $\kappa \equiv \sqrt{\xi^2/c^2 + k^2}$ and $R_{p,p'}(\mathbf{k}, \mathbf{k}')$ dependent on material properties at freq. $i\xi$

Specular/non specular scattering



In order to treat a general rough or corrugated surface, we make a perturbative expansion in powers of $h(x,y)$

$$\mathcal{R} = \mathcal{R}^{(0)} + \mathcal{R}^{(1)} + \dots$$



□ Specular reflection:

$$\langle \mathbf{k}, p | \mathcal{R}^{(0)} | \mathbf{k}', p' \rangle = (2\pi)^2 \delta^{(2)}(\mathbf{k} - \mathbf{k}') \delta_{p,p'} r_p(\mathbf{k}, \xi)$$

Fresnel coefficients $r_{\text{TE}} = \frac{\kappa - \kappa_t}{\kappa + \kappa_t}$ $r_{\text{TM}} = \frac{\epsilon(i\xi)\kappa - \kappa_t}{\epsilon(i\xi)\kappa + \kappa_t}$ ($\kappa_t = \sqrt{\epsilon(i\xi)\xi^2/c^2 + k^2}$)

□ Non-specular reflection:

$$\langle \mathbf{k}, p | \mathcal{R}^{(1)} | \mathbf{k}', p' \rangle = R_{p,p'}(\mathbf{k}, \mathbf{k}') H(\mathbf{k} - \mathbf{k}')$$

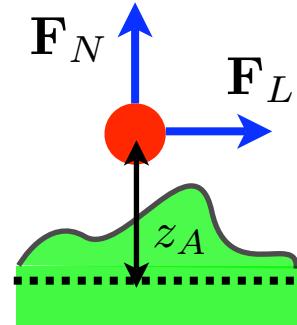


Fourier transform of $h(x,y)$



The non-specular reflection matrices depend on the geometry and material properties.

Lateral Casimir-Polder force



$$U_{\text{CP}} = U_{\text{CP}}^{(0)}(z_A) + U_{\text{CP}}^{(1)}(z_A, x_A)$$

■ **Normal CP force:**

$$U_{\text{CP}}^{(0)}(z_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{1}{2\kappa} \sum_p \hat{\epsilon}_p^+ \cdot \hat{\epsilon}_p^- r_p(\mathbf{k}, \xi) e^{-2\kappa z_A}$$

■ **Lateral CP force:**

$$U_{\text{CP}}^{(1)}(z_A, x_A) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k} \cdot \mathbf{r}_A} g(\mathbf{k}, z_A) H(\mathbf{k})$$

Response function g:

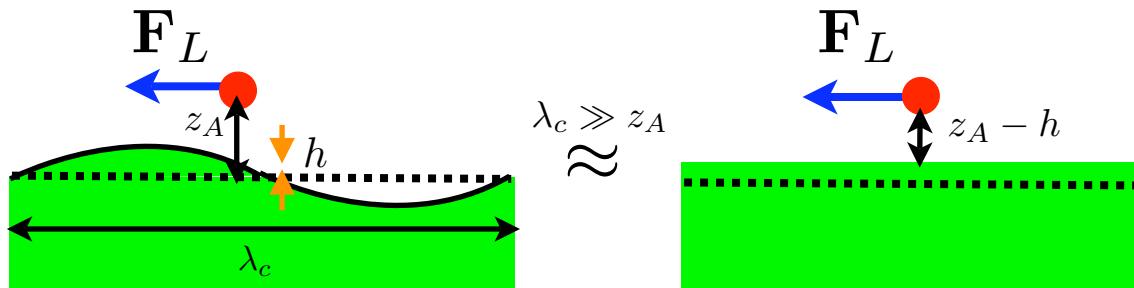
$$g(\mathbf{k}, z_A) = \frac{\hbar}{c^2 \epsilon_0} \int_0^\infty \frac{d\xi}{2\pi} \xi^2 \alpha(i\xi) \int \frac{d^2\mathbf{k}'}{(2\pi)^2} a_{\mathbf{k}', \mathbf{k}' - \mathbf{k}}(z_A, \xi)$$

$$a_{\mathbf{k}', \mathbf{k}''} = \sum_{p', p''} \hat{\epsilon}_{p'}^+ \cdot \hat{\epsilon}_{p''}^- \frac{e^{-(\kappa' + \kappa'') z_A}}{2\kappa''} R_{p', p''}(\mathbf{k}', \mathbf{k}'')$$

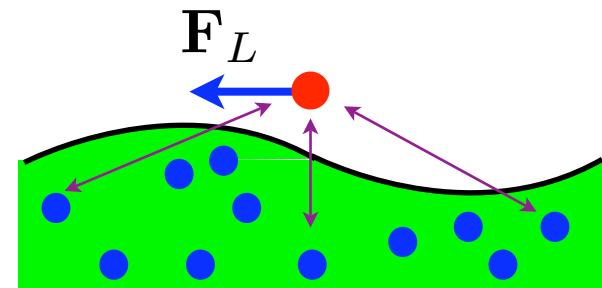
Our approach is perturbative in $h(x,y)$, which should be the smallest length scale in the problem $h \ll z_A, \lambda_c, \lambda_A, \lambda_0$

Approx. methods: PFA & PWS

Proximity Force Approximation (PFA)

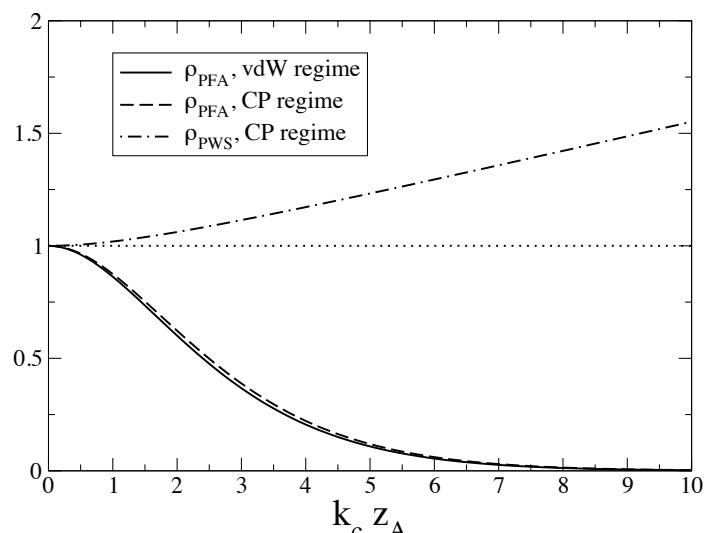


Pair-wise Summation (PWS)



Deviations from PFA and PWS

$$\rho_{\text{PFA}} = \frac{g(k_c, z_A)}{g(0, z_A)} \quad \rho_{\text{PWS}} \equiv \frac{g(k_c, z_A)}{g_{\text{PWS}}(k_c, z_A)}$$



Example:

atom-surface distance $z_A = 2\mu\text{m} \gg \lambda_A$

corrugation wavelength $\lambda_c = 3.5\mu\text{m}$

→ $\rho_{\text{PFA}} \approx 30\%$ $\rho_{\text{PWS}} \approx 115\%$

PFA largely overestimates the lateral CP force
PWS underestimates the lateral CP force

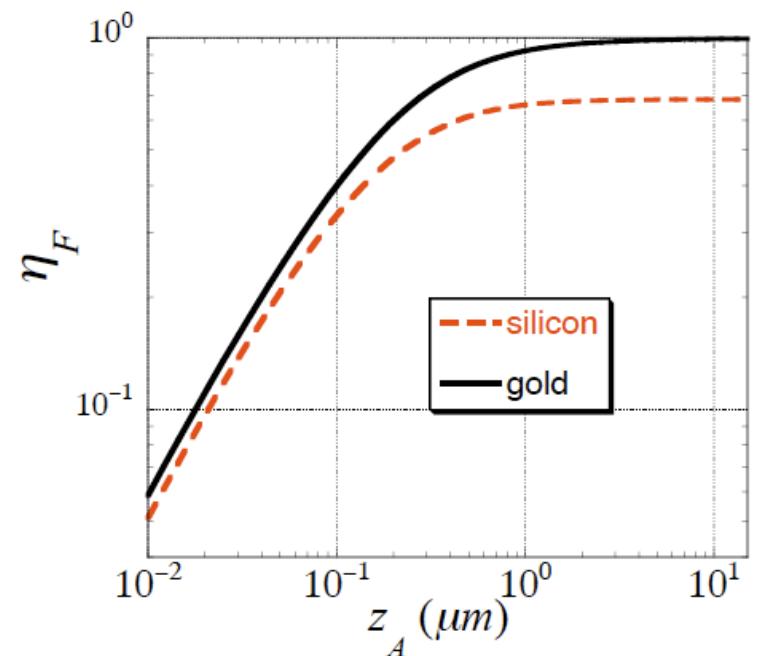
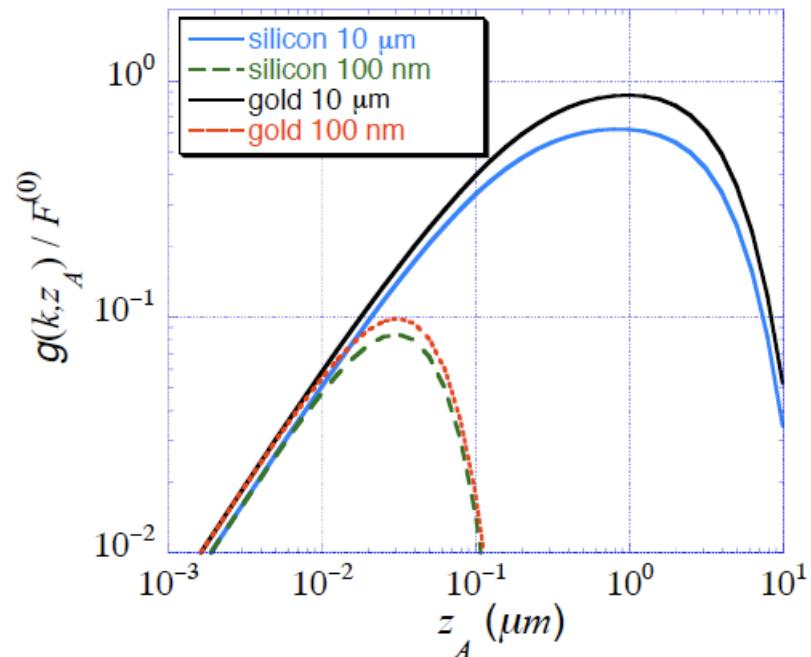
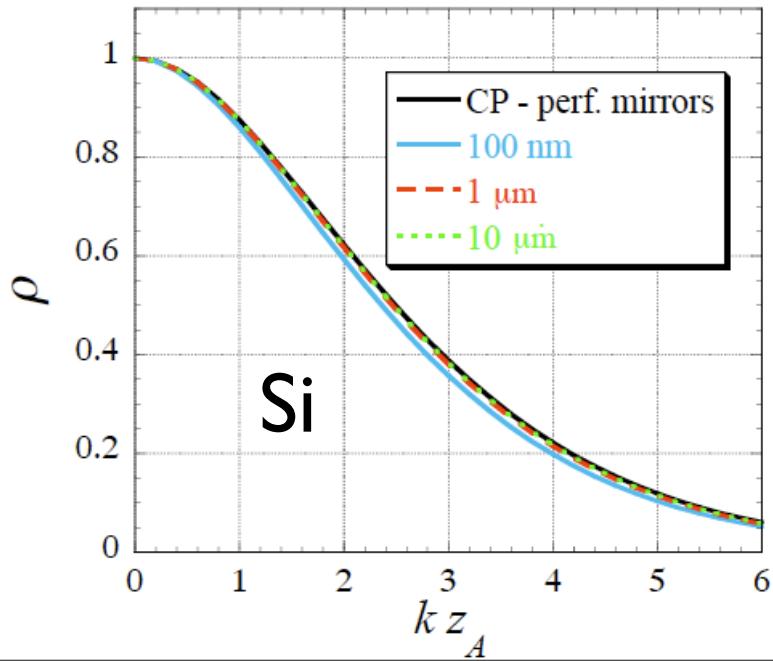
Disentangling geometry and real materials effects

$$g(k, z) = \rho(k, z) \eta_F F_{\text{CP}}^{(0)}$$

effects of
materials

effects of
geometry

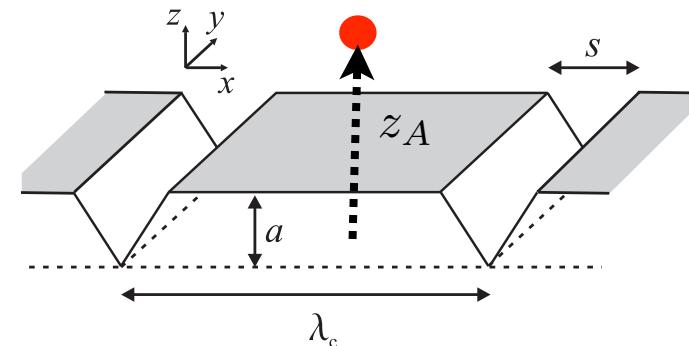
$$\rho_{\text{CP}}^{\text{perf}}(k, z_A) = e^{-kz_A} \left[1 + kz_A + \frac{16(kz_A)^2}{45} + \frac{(kz_A)^3}{45} \right]$$



CP energy for grooved surface

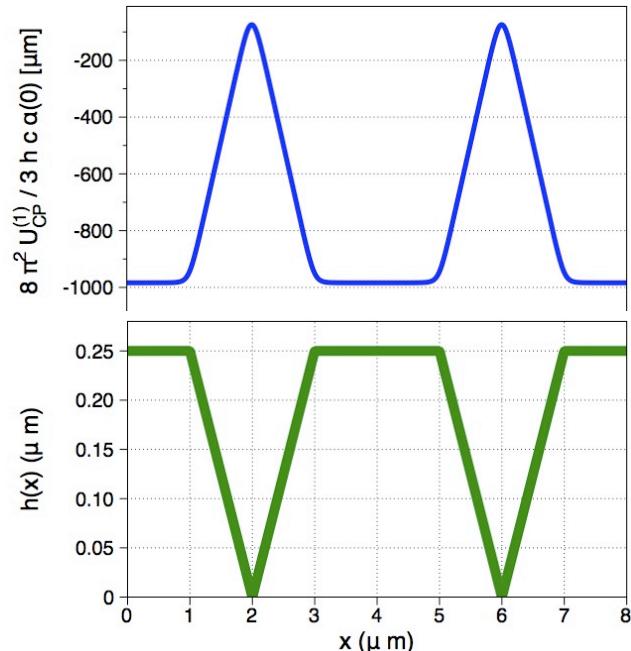
- Surface profile for periodical grooved corrugation

$$h(x) = a \left(1 - \frac{s}{2\lambda_c}\right) + \frac{2a\lambda_c}{\pi^2 s} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1 - \cos(n\pi s/\lambda_c)}{n^2} \cos\left(\frac{2\pi n x}{\lambda_c}\right)$$

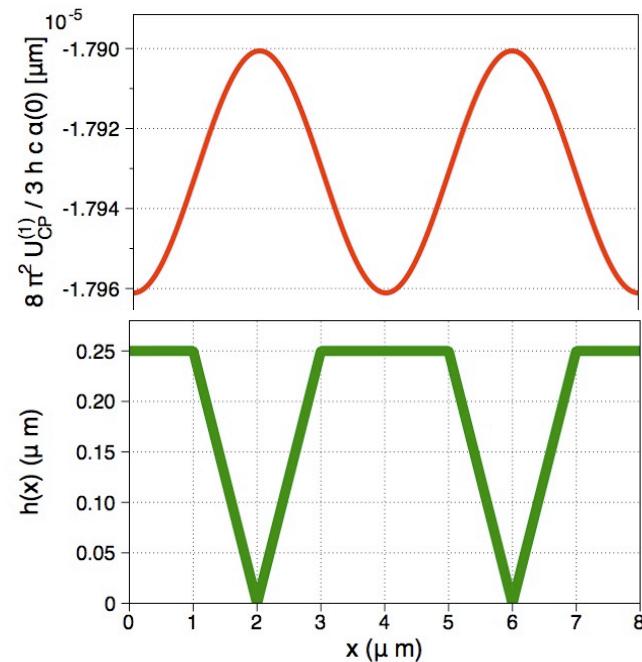


- Single-atom lateral CP energy: it can be easily calculated using that the first order lateral CP energy $U_{\text{CP}}^{(1)}(\mathbf{R}_A) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}\cdot\mathbf{r}_A} g(\mathbf{k}, z_A) H(\mathbf{k})$ is linear in $H(\mathbf{k})$

$$k_c z_A = 0.3$$



$$k_c z_A = 10$$



Experimental proposals

- BEC “cantilever” to measure Casimir frequency shifts
- BEC Bragg spectroscopy to measure the Casimir lateral energy profile

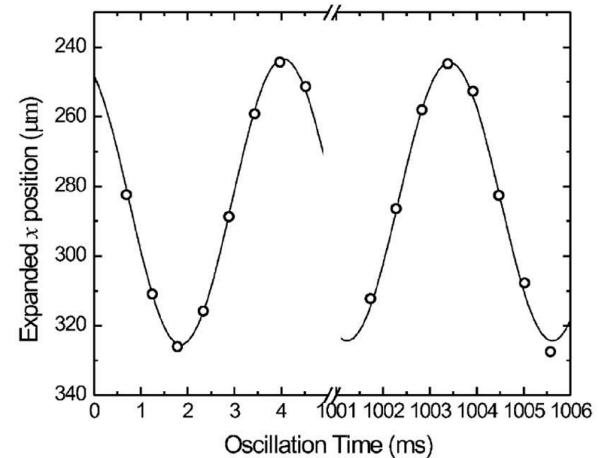
BEC as a “cantilever”



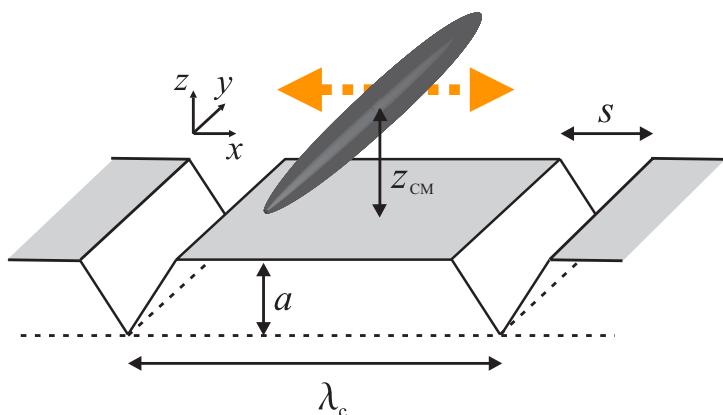
BEC oscillator

- The normal component of Casimir-Polder force $U_{\text{CP}}^{(0)}(z)$ shifts the **normal dipolar oscillation frequency** of a BEC trapped above a surface

Antezza et al (2004) Cornell et al (2005, 2007)



- In order to measure the lateral component $U_{\text{CP}}^{(1)}(x, z)$, a cigar-shaped BEC could be trapped parallel to the corrugation lines, and the **lateral dipolar oscillation** measured as a function of time



$$V(\mathbf{r}) = V_{\text{ho}}(\mathbf{r}) + U_{\text{CP}}(\mathbf{r})$$

$$V_{\text{ho}}(\mathbf{r}) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \quad \omega_y \ll \omega_x = \omega_z$$

Lateral frequency shift:

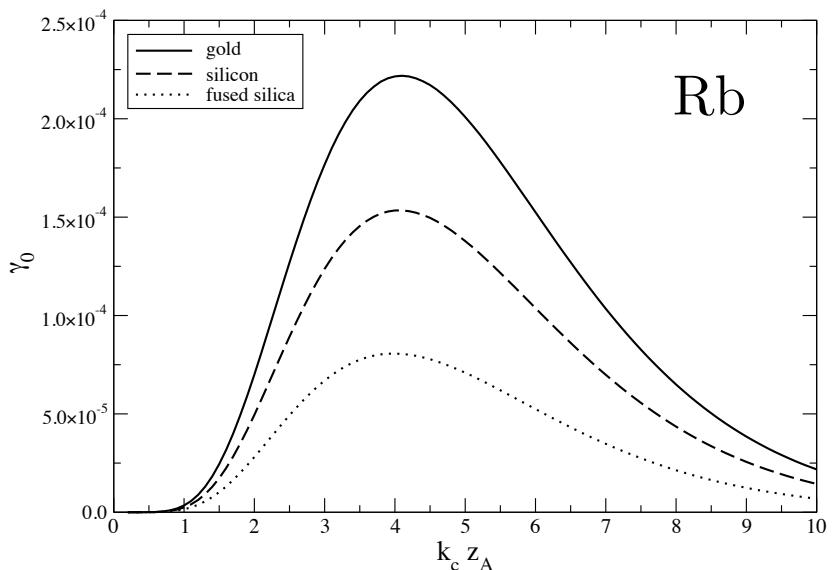
$$\omega_{x,\text{CM}}^2 = \omega_x^2 + \frac{1}{m} \int dx dz n_0(x, z) \frac{\partial^2}{\partial x^2} U_{\text{CP}}^{(1)}(x, z)$$

Single-atom/BEC frequency shift

$$\gamma_0 \equiv \frac{\omega_{x,CM} - \omega_x}{\omega_x}$$



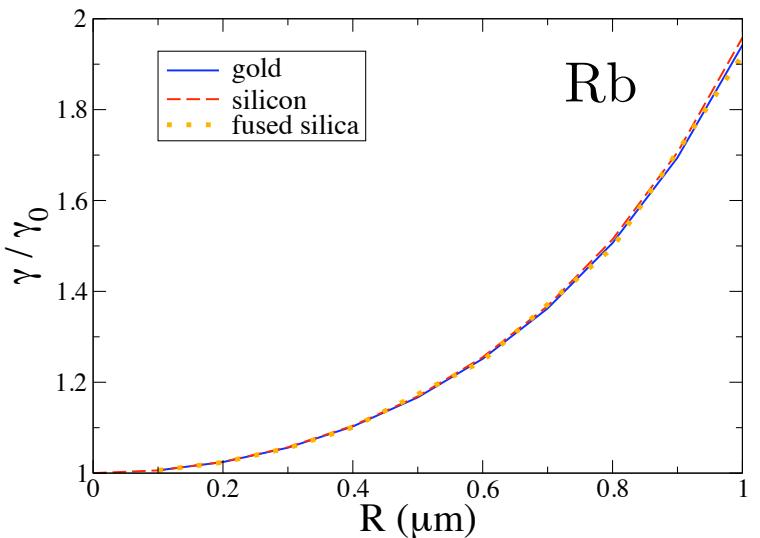
Single-atom lateral freq. shift



$$\begin{aligned}\omega_x / 2\pi &= 229 \text{ Hz} \\ z_{CM} &= 2 \mu\text{m} \\ \lambda_c &= 4 \mu\text{m} \\ a &= 250 \text{ nm} \\ s &= \lambda_c / 2\end{aligned}$$



Single-atom / BEC comparison

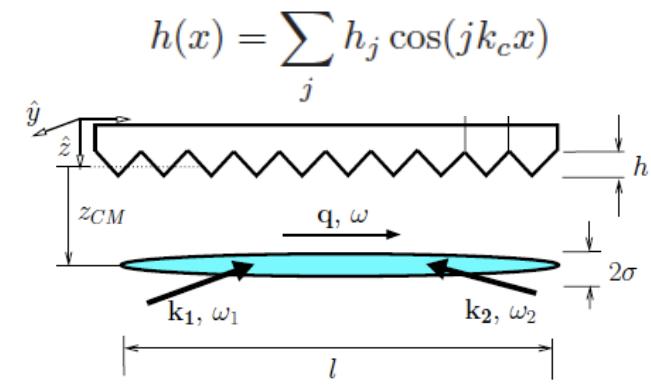


Given the reported sensitivity $\gamma = 10^{-5} - 10^{-4}$ for relative frequency shifts from E. Cornell's experiment, we expect that beyond-PFA lateral CP forces on a BEC above a plateau of a periodically grooved silicon surface should be detectable for distances $z_{CM} < 3 \mu\text{m}$, groove period $\lambda_c = 4 \mu\text{m}$, groove amplitude $a = 250 \text{ nm}$, and a BEC radius of, say, $R \approx 1 \mu\text{m}$

Casimir-modified BEC low energy spectrum

- Mean field BEC dynamics given by the Gross-Pitaevskii equation for the condensate wave-function φ

$$\begin{aligned} i\hbar\partial_t\varphi &= -(\hbar^2/2m)\nabla^2\varphi + [U_N(z) + U_L(x, z)]\varphi \\ &\quad + (m/2)(\omega_r^2 r^2 + \omega_x^2 x^2)\varphi + g|\varphi|^2\varphi, \end{aligned}$$



$$h(x) = \sum_j h_j \cos(jk_c x)$$

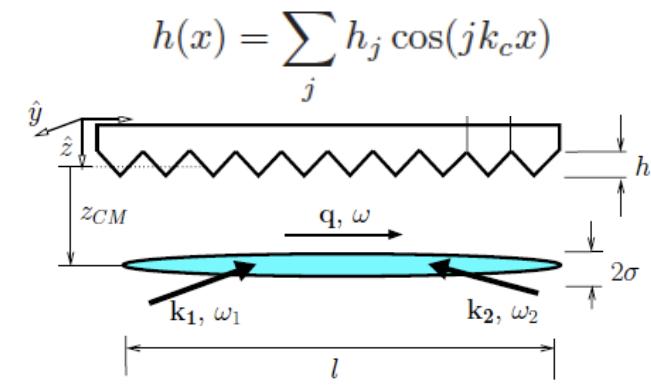
$$U(x, y, z) = U_N(z) + U_L(x, z)$$

$$U_L^{(1)}(x_A, z_A) = \sum_{j=0}^{\infty} h_j \cos(jk_c x_A) g(jk_c, z_A)$$

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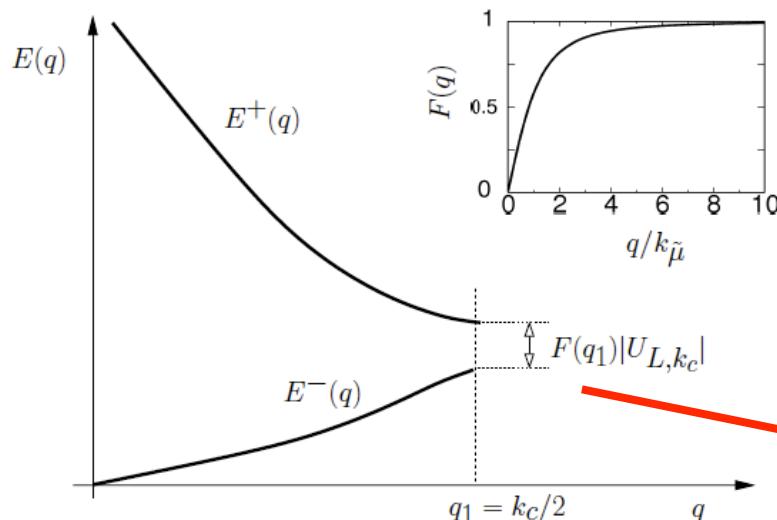
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$$U(x, y, z) = U_N(z) + U_L(x, z)$$

$$U_L^{(1)}(x_A, z_A) = \sum_{j=0}^{\infty} h_j \cos(jk_c x_A) g(jk_c, z_A)$$

- Quantum fluctuations given by the Bogoliubov spectrum + Casimir modifications:



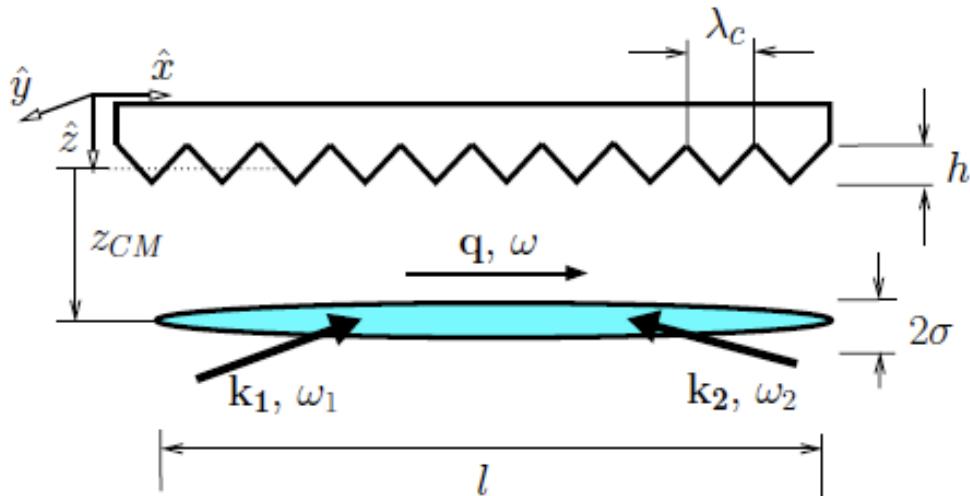
$$E(q) = E_B(q) + \delta E_{CP}(q)$$

$$E_B(q) = \sqrt{(\hbar^2 q^2/2m)(\hbar^2 q^2/2m + 2\tilde{\mu})}$$

$$\tilde{\mu} = \mu - \hbar\omega_r - U_N(z_{cm})$$

CP opens energy gaps

BEC Bragg spectroscopy



Observable: total momentum P_X transferred to the BEC

Two Bragg photons set-up

$$\mathbf{q} = q\hat{x} = \mathbf{k}_1 - \mathbf{k}_2 ; \omega = \omega_1 - \omega_2.$$

$$\begin{aligned} \frac{dP_X}{dt} = & -m\omega_x^2 X + \sum_{n,i} U_{L,nk_c}(nk_c) \langle \sin(nk_c x_i) \rangle + \\ & + \frac{N\hbar q V_B^2}{2} \int d\omega' [S(q, \omega') - S(-q, -\omega')] \\ & \times \frac{\sin([\omega - \omega']t)}{\omega - \omega'}. \end{aligned}$$

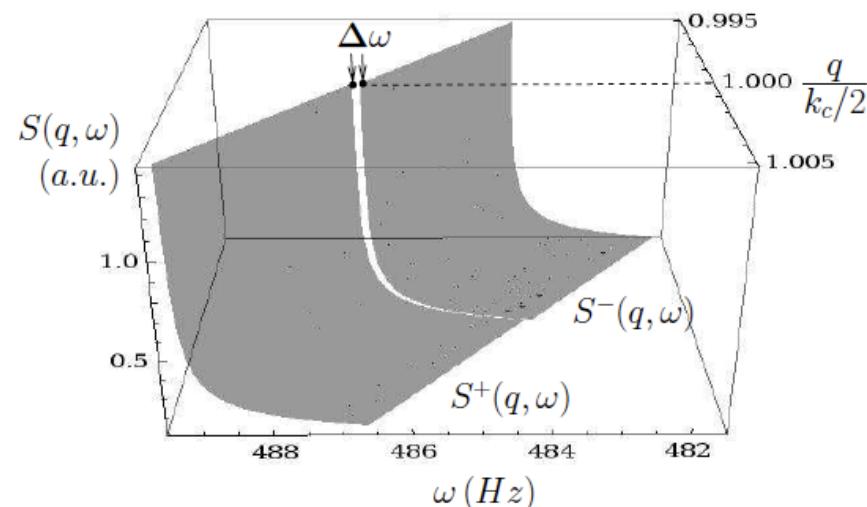
Dynamic structure factor:

$$S(q, \omega) = \frac{N\hbar^2 q^2}{2mE_B(q)} \delta(\hbar\omega - E_B(q))$$

$$S \rightarrow S + \delta S_{CP}(q, \omega)$$

[without CP force]

[with CP force]



Is this feasible?

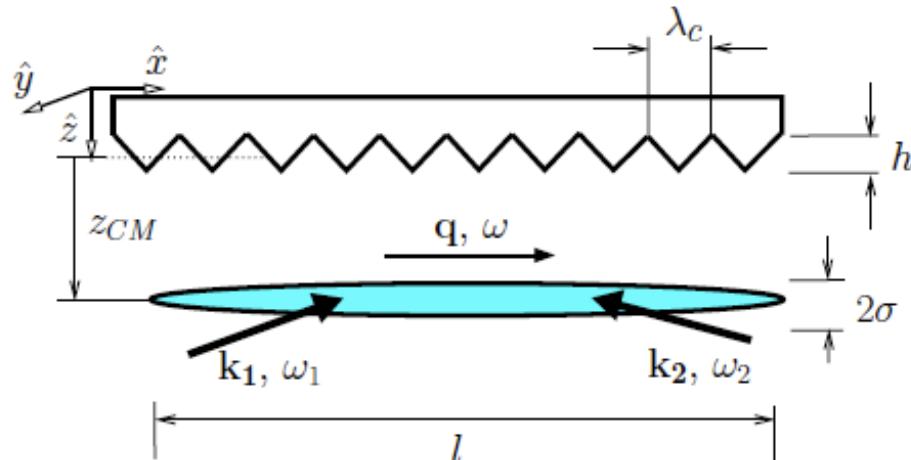
Parameters of the BEC (^{87}Rb)

$$N = 10^4 \text{ atoms}$$

$$\omega_r/2\pi = 2.7\text{kHz} \quad \omega_x/2\pi = 0.83 \text{ Hz}$$

$$\sigma = 0.2\mu\text{m} \quad l/2 = 408\mu\text{m}$$

$$E_B(q_1) = \hbar 485\text{Hz}$$



$$\lambda_c = 2\pi/k_c = 9.75\mu\text{m}$$

Parameters of the surface

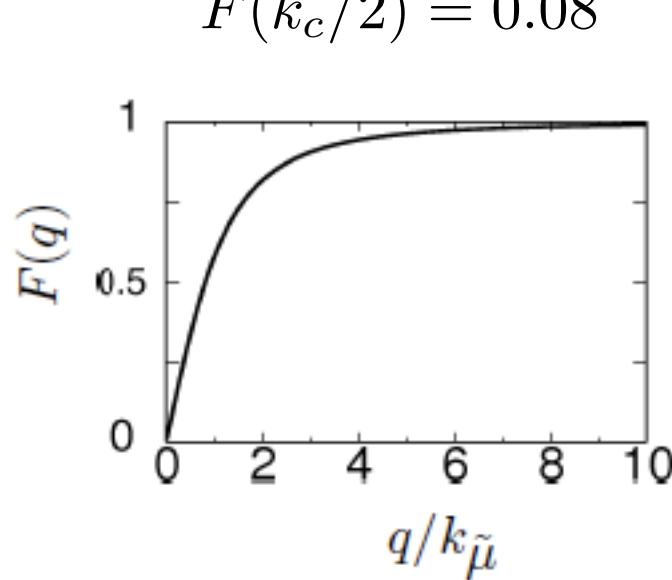
$$h = 1\mu\text{m} \quad z_{cm} = 3\mu\text{m}$$

$$g_{\text{CP}}^{\text{perf}}(k, z) = -\frac{3\hbar c \alpha(0)}{8\pi^2 \epsilon_0 z^5} e^{-\mathcal{Z}} (1 + \mathcal{Z} + 16\mathcal{Z}^2/45 + \mathcal{Z}^3/45)$$

$$\mathcal{Z} = k_c z \quad \alpha(0)/\epsilon_0 = 47.3 \times 10^{-30} \text{ m}^3$$

$$U_{L,k_c}^{(1)} = h g_{\text{CP}}^{\text{perf}}(k_c, z_{cm}) \approx \begin{cases} \hbar 1.26\text{Hz} & [\text{Au}] \\ \hbar 0.98\text{Hz} & [\text{Si}] \end{cases}$$

$$U_{L,k_c}^{(1)} F(k_c/2) \approx \hbar 0.1 \text{ Hz}$$



Summary

- Novel cold atoms techniques open a promising way of investigating nontrivial geometrical effects on quantum vacuum
- Important feature of atoms: they can be used as local probes of quantum vacuum fluctuations
- Non-trivial, beyond-PFA effects should be measurable using a BEC as a vacuum field sensor with available technology

For more details see:

Phys. Rev. Lett. 100, 040405 (2008)

J. Phys. A 41, 164028 (2008)

arXiv: 0902.3235

arXiv: 0904.0238